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# TRACKING THE SLOWDOWN IN LONG-RUN GDP GROWTH

Juan Antolin-Diaz, Thomas Drechsel, and Ivan Petrella\*

**Abstract**—Using a dynamic factor model that allows for changes in both the long-run growth rate of output and the volatility of business cycles, we document a significant decline in long-run output growth in the United States. Our evidence supports the view that most of this slowdown occurred prior to the Great Recession. We show how to use the model to decompose changes in long-run growth into its underlying drivers. At low frequencies, a decline in the growth rate of labor productivity appears to be behind the recent slowdown in GDP growth for both the United States and other advanced economies. When applied to real-time data, the proposed model is capable of detecting shifts in long-run growth in a timely and reliable manner.

## I. Introduction

*The global recovery has been disappointing. . . . Year after year we have had to explain from mid-year on why the global growth rate has been lower than predicted as little as two quarters back.*

Stanley Fischer, August 2014

THE slow pace of the recovery from the Great Recession of 2007–2009 has prompted questions about whether the long-run growth rate of GDP in advanced economies is lower now than it has been on average over the past decades (see, e.g., Fernald, 2014; Gordon, 2014b; Summers, 2014). Indeed, forecasts of U.S. and global real GDP growth have been persistently too optimistic in the years after the recession (2010–2015).<sup>1</sup> As Orphanides (2003) emphasized, real-time misperceptions about the long-run growth of the economy can play a large role in monetary policy mistakes. Moreover, small changes in assumptions about the long-run growth rate of output can have large implications on fiscal sustainability calculations (Auerbach, 2011). This calls for a framework that takes the uncertainty about long-run growth seriously and can inform decision making in real time. In this paper, we present a dynamic factor model (DFM) that allows

for gradual changes in the mean and the variance of real output growth. By incorporating a broad panel of economic activity indicators, DFMs are capable of precisely estimating the cyclical comovement in macroeconomic data in a real-time setting. Our model exploits this to track changes in the long-run growth rate of real GDP in a timely and reliable manner, separating them from their cyclical counterpart.<sup>2</sup>

The evidence of a decline in long-run U.S. growth is accumulating, as documented by the recent growth literature such as Fernald and Jones (2014). Lawrence Summers and Robert Gordon have articulated a particularly pessimistic view of long-run growth that contrasts with the optimism prevailing before the Great Recession (see Jorgenson, Ho, & Stiroh, 2006). To complement this evidence, we start our analysis by presenting the results of two popular structural break tests proposed by Nyblom (1989) and Bai and Perron (1998). Both suggest that a possible shift in the mean of U.S. real GDP growth exists, the latter approach suggesting that a break probably occurred in the early part of the 2000s. However, sequential testing using real-time data reveals that the break would not have been detected at conventional significance levels until as late as mid-2012, highlighting the problems of conventional break tests for real-time analysis (see also Benati, 2007). To address this issue, we introduce two novel features into an otherwise standard DFM of real activity data. First, we allow the mean of real GDP growth, and possibly other series, to drift gradually over time. As Cogley (2005) emphasized, if the long-run output growth rate is not constant, it is optimal to give more weight to recent data when estimating its current state. By taking a Bayesian approach, we can combine our prior beliefs about the rate at which the past information should be discounted with the information contained in the data. We also characterize the uncertainty around estimates of long-run growth, taking into account both filtering and parameter uncertainty. Second, we allow for stochastic volatility (SV) in the innovations to both factors and idiosyncratic components. Given our interest in studying the entire postwar period, the inclusion of SV is essential to capture the substantial changes in the volatility of output that have taken place in this sample, such as the “Great Moderation” first reported by Kim and Nelson (1999a) and McConnell and Perez-Quiros (2000), as well as the cyclical volatility of macroeconomic volatility as documented by Jurado, Ludvigson and Ng (2015).

When applied to U.S. data, our model concludes that long-run GDP growth declined meaningfully during the 2000s and currently stands at about 2%, more than 1 percentage point lower than the postwar average. The results are

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<sup>1</sup> For instance, Federal Open Market Committee (FOMC) projections since 2009 expected U.S. growth to accelerate substantially, only to downgrade the forecast back to 2% throughout the course of the subsequent year. An analysis of forecasts produced by international organizations and private sector economists reveals the same pattern. See Pain et al. (2014), for a retrospective.

<sup>2</sup> Throughout this paper, our concept of the long run refers to changes in growth that are permanent in nature, that is, do not mean-revert, as in Beveridge and Nelson (1981). In practice, this should be thought of as frequencies lower than the business cycle.

supportive of a gradual decline rather than a discrete break. Since in-sample results obtained with revised data often underestimate the uncertainty that policymakers face in real time, we repeat the exercise using real-time vintages of data. The model detects the fall from the beginning of the 2000s onward, and by the summer of 2010, it reaches the significant conclusion that a decline in long-run growth is behind the slow recovery, well before the structural break tests become conclusive.

We also investigate the performance of the model in “nowcasting” short-term developments in GDP. Since the seminal contributions of Giannone, Reichlin, and Small (2008) DFMs have become the standard tool for this purpose. Interestingly, our analysis shows that standard DFM forecasts revert very quickly to the unconditional mean of GDP, so taking into account the variation in long-run GDP growth substantially improves point and density GDP forecasts even at very short horizons.

Finally, we extend our model in order to disentangle the drivers of secular fluctuations of GDP growth. Edge, Laubach, and Williams (2007) emphasize the relevance as well as the difficulty of tracking permanent shifts in productivity growth in real time. In our framework, long-run output growth can be decomposed into labor productivity and labor input trends. The results of this decomposition exercise point to a slowdown in labor productivity as the main driver of recent weakness in GDP growth. Applying the model to other advanced economies, we provide evidence that the weakening in labor productivity appears to be a global phenomenon.

Our work is closely related to two strands of literature. The first one encompasses papers that allow for structural changes within the DFM framework. Del Negro and Otrok (2008) model time variation in factor loadings and volatilities, while Marcellino, Porqueddu, and Venditti (2016) show that the addition of SV improves the performance of the model for short-term forecasting of euro-area GDP. Acknowledging the importance of allowing for time variation in the means of the variables, Stock and Watson (2012) prefilter their data set in order to remove any low-frequency trends from the resulting growth rates using a biweight local mean. In his comment to their paper, Sims (2012) suggests explicitly modelling, rather than filtering out, these long-run trends, and emphasizes the importance of evolving volatilities for describing and understanding macroeconomic data. We see our paper as extending the DFM literature and, in particular, its application to tracking GDP, in the direction suggested by Chris Sims. The second strand of related literature takes a similar approach to decomposing long-run GDP growth into its drivers, in particular Gordon (2010, 2014a) and Reifschneider, Wescher, and Wilcox (2013). Relative to these studies, we emphasize the importance of using a broader information set, as well as a Bayesian approach, which allows using priors to inform the estimate of long-run growth and characterize the uncertainty around the estimate stemming from both filtering and parameter uncertainty.

The remainder of this paper is organized as follows. Section II presents preliminary evidence of a slowdown in long-run U.S. GDP growth. Section III discusses the implications of time-varying long-run output growth and volatility for DFMs and presents our model. Section IV applies the model to U.S. data and documents the decline in long-run growth. The implications for tracking GDP in real time, as well as the key advantages of our methodology, are discussed. Section V decomposes the changes in long-run output growth into its underlying drivers. Section VI concludes.

## II. Preliminary Evidence

The literature on economic growth favors a view of the long-run growth rate as a process that evolves over time. It is by now widely accepted that a slowdown in productivity and long-run output growth occurred in the early 1970s, and that accelerating productivity in the IT sector led to a boom in the late 1990s. In contrast, in the context of econometric modeling, the possibility that long-run growth is time varying is the source of a long-standing controversy. In their seminal contribution, Nelson and Plosser (1982) model the (log) level of real GDP as a random walk with drift. This implies that after first-differencing, the resulting growth rate fluctuates around a constant mean, an assumption still embedded in many econometric models. After the slowdown in productivity became apparent in the 1970s, many researchers such as Clark (1987) modeled the drift term as an additional random walk, implying that the level of GDP is integrated of order two. The latter assumption would also be consistent with the local linear trend model of Harvey (1985), the Hodrick and Prescott (1997) filter, and Stock and Watson’s (2012) practice of removing a local biweight mean from the growth rates before estimating a DFM. The  $I(2)$  assumption is nevertheless controversial since it implies that the growth rate of output can drift without bound. Consequently, papers such as Perron and Wada (2009) have modeled the growth rate of GDP as stationary around a trend with one large break around 1973.

Ever since the Great Recession of 2007–2009, US real GDP has grown well below its postwar average, once again raising the question whether its mean may have declined. There are two popular strategies that could be followed from a frequentist perspective to detect parameter instability or the presence of breaks in the mean growth rate. The first one is Nyblom’s (1989) L-test, as described in Hansen (1992), which tests the null hypothesis of constant parameters against the alternative that the parameters follow a martingale. Modeling real GDP growth as an AR(1) over the sample 1947–2015, this test rejects the stability of the constant term at the 10% significance level.<sup>3</sup> The second

<sup>3</sup>The same result holds for an AR(2) specification. In both cases, the stability of the autoregressive coefficients cannot be rejected, whereas the stability of the variance is rejected at the 1% level. Section B of the online appendix provides the full results of both tests applied in this section.

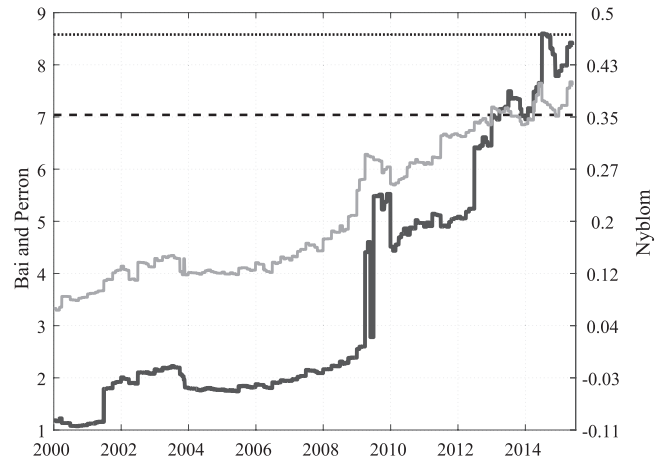
commonly used approach, which can determine the number and timing of multiple discrete breaks, is the Bai and Perron (1998) test. This test finds evidence in favor of a single break in the mean of U.S. real GDP growth at the 10% level. The most likely break date is in the second quarter of 2000. In related research, Fernald (2014) provides evidence for breaks in labor productivity in 1973:Q2, 1995:Q3, and 2003:Q1 and links the latter two to developments in the IT sector. From a Bayesian perspective, Luo and Startz (2014) calculate the posterior probability of a single break and find the most likely break date to be 2006:Q1 for the full postwar sample and 1973:Q1 for a sample excluding the 2000s.

These results indicate that substantial evidence for a recent change in the mean of U.S. GDP growth has built up. However, the strategy of applying conventional tests and introducing deterministic breaks into econometric models is not satisfactory for the purposes of real-time decision making. In fact, the detection of change in the mean of GDP growth can arrive with substantial delay. To demonstrate this, a sequential application of the Nyblom (1989) and Bai and Perron (1998) tests using real-time data is presented in figure 1. The evolution of the test statistics in real time reveals that a break would not have been detected at the 10% significance levels until as late as mid-2012, more than ten years later than the actual break date suggested by Bai and Perron's (1998) procedure. The Nyblom (1989) test, which is designed to detect gradual change rather than a discrete break, becomes significant at roughly the same time. This lack of timeliness highlights the importance of an econometric framework capable of quickly adapting to changes in long-run growth as new information arrives.

### III. Econometric Framework

DFMs in the spirit of Sargent and Sims (1977) and Forni et al. (2009) capture the idea that a small number of unobserved factors drives the comovement of a possibly large number of macroeconomic time series, each of which may be contaminated by measurement error or other sources of idiosyncratic variation. Their theoretical appeal, as well as their ability to parsimoniously model large data sets, have made them a workhorse of empirical macroeconomics. Giannone et al. (2008) have pioneered the use of DFMs to produce current-quarter forecasts ("nowcasts") of GDP growth by exploiting more timely monthly indicators and the factor structure of the data. Given the widespread use of DFMs to track GDP in real time, this paper aims to make these models robust to changes in long-run growth. We do so by introducing two novel features into the DFM framework. First, we allow the long-run growth rate of real GDP, and possibly other series, to vary over time. Second, we allow for stochastic volatility (SV) in the innovations to both factors and idiosyncratic components, given our interest in studying the entire postwar period for which drastic changes in volatility have been documented. With these changes, the DFM proves to be a powerful tool to detect changes in long-run

FIGURE 1.—REAL-TIME TEST STATISTICS OF THE NYBLOM AND BAI-PERRON TESTS



The gray and black solid lines are the values of the test statistics obtained from sequentially reapplying the Nyblom (1989) and Bai and Perron (1998) tests in real time as new National Accounts vintages are being published. In both cases, the sample starts in 1947:Q2, and the test is reapplied for every new data release occurring after the beginning of 2000. The dotted and dashed horizontal lines represent, respectively, the 5% and 10% critical values corresponding to the two tests.

growth. The information contained in a broad panel of activity indicators facilitates the timely decomposition of real GDP growth into persistent long-run movements, cyclical fluctuations, and short-lived noise.

#### A. The Model

Let  $\mathbf{y}_t$  be an  $n \times 1$  vector of observable macroeconomic time series, and let  $\mathbf{f}_t$  denote a  $k \times 1$  vector of latent common factors. It is assumed that  $n \gg k$  (i.e., the number of observables is much larger than the number of factors). Formally,

$$\mathbf{y}_t = \mathbf{c}_t + \mathbf{\Lambda} \mathbf{f}_t + \mathbf{u}_t, \quad (1)$$

where  $\mathbf{\Lambda}$  contains the loadings on the common factors and  $\mathbf{u}_t$  is a vector of idiosyncratic components.<sup>4</sup> Shifts in the long-run mean of  $\mathbf{y}_t$  are captured by time variation in  $\mathbf{c}_t$ . In principle, one could allow time-varying intercepts in all or a subset of the variables in the system. Moreover, time variation in a given series could be shared by other series.  $\mathbf{c}_t$  is therefore flexibly specified as

$$\mathbf{c}_t = \begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{c} \end{bmatrix} \begin{bmatrix} \mathbf{a}_t \\ 1 \end{bmatrix}, \quad (2)$$

where  $\mathbf{a}_t$  is an  $r \times 1$  vector of time-varying means,  $\mathbf{B}$  is an  $m \times r$  matrix that governs how the time variation affects the corresponding observables, and  $\mathbf{c}$  is an  $(n - m) \times 1$  vector of constants. In our baseline specification,  $\mathbf{a}_t$  will be a scalar capturing time variation in long-run real GDP growth, which is shared by real consumption growth, so that  $r = 1, m = 2$ .

<sup>4</sup>The model can be easily extended to include lags of the factor in the measurement equation (see appendix J).

A detailed discussion of this and additional specifications of  $\mathbf{c}_t$  is provided in section IIIB.

Throughout the paper, we focus on the case of a single dynamic factor by setting  $k = 1$  (i.e.,  $\mathbf{f}_t = f_t$ ).<sup>5</sup> The laws of motion of the latent factor and the idiosyncratic components are

$$(1 - \phi(L))f_t = \sigma_{\varepsilon_t}\varepsilon_t, \quad (3)$$

$$(1 - \rho_i(L))u_{i,t} = \sigma_{\eta_{i,t}}\eta_{i,t}, \quad i = 1, \dots, n, \quad (4)$$

where  $\phi(L)$  and  $\rho_i(L)$  denote polynomials in the lag operator of order  $p$  and  $q$ , respectively. The idiosyncratic components are cross-sectionally orthogonal and are assumed to be uncorrelated with the common factor at all leads and lags:  $\varepsilon_t \stackrel{iid}{\sim} N(0, 1)$  and  $\eta_{i,t} \stackrel{iid}{\sim} N(0, 1)$ .

Finally, the dynamics of the model's time-varying parameters are specified to follow driftless random walks,

$$a_{j,t} = a_{j,t-1} + v_{a_{j,t}}, \quad v_{a_{j,t}} \stackrel{iid}{\sim} N(0, \omega_{a,j}^2) \quad j = 1, \dots, r, \quad (5)$$

$$\log \sigma_{\varepsilon_t} = \log \sigma_{\varepsilon_{t-1}} + v_{\varepsilon,t}, \quad v_{\varepsilon,t} \stackrel{iid}{\sim} N(0, \omega_{\varepsilon}^2), \quad (6)$$

$$\log \sigma_{\eta_{i,t}} = \log \sigma_{\eta_{i,t-1}} + v_{\eta_{i,t}}, \quad v_{\eta_{i,t}} \stackrel{iid}{\sim} N(0, \omega_{\eta,i}^2) \quad i = 1, \dots, n, \quad (7)$$

where  $a_{j,t}$  are the  $r$  time-varying elements in  $\mathbf{a}_t$  and  $\sigma_{\varepsilon_t}$  and  $\sigma_{\eta_{i,t}}$  capture the SV of the innovations to factor and idiosyncratic components. Our motivation for specifying the time-varying parameters as random walks is similar to Primiceri (2005). While in principle it is unrealistic to model real GDP growth as a process that could wander in an unbounded way, as long as the variance of the process is small and the drift is considered to be operating for a finite period of time, the assumption is innocuous. Moreover, modeling a trend as a random walk is more robust to misspecification when the actual process is instead characterized by discrete breaks, whereas models with discrete breaks might not be robust to the true process being a random walk.<sup>6</sup> Finally, the random walk assumption also has the desirable feature that, unlike stationary models, confidence bands around forecasts of real GDP growth increase with the forecast horizon, reflecting uncertainty about the possibility of future breaks or drifts in long-run growth.

Note that a standard DFM is usually specified under two assumptions. First, the original data have been differenced

appropriately so that both the factor and the idiosyncratic components can be assumed to be stationary. Second, it is assumed that the innovations in the idiosyncratic and common components are i.i.d. In equations 1 to 7 we have relaxed these assumptions to allow for two novel features: a stochastic trend in the mean of selected series and SV. By shutting down these features, we can recover the specifications previously proposed in the literature, which are nested in our framework. We obtain the DFM with SV of Marcellino et al. (2016) if we shut down time variation in the intercepts of the observables, that is, set  $r = m = 0$  and  $\mathbf{c}_t = \mathbf{c}$ . If we further shut down the SV, that is, set  $\omega_{a,j}^2 = \omega_{\varepsilon}^2 = \omega_{\eta,i}^2 = 0$ , we obtain the specification of Banbura, Giannone, and Reichlin (2010) and Banbura and Modugno (2014).

## B. A Baseline Specification for Long-Run Growth

Equations (1) and (2) allow for stochastic trends in the mean of all or a subset of selected observables in  $\mathbf{y}_t$ . This paper focuses on tracking changes in the long-run growth rate of real GDP. For this purpose, the simplest specification of  $\mathbf{c}_t$  is to include a time-varying intercept only in GDP and to set  $\mathbf{B} = \mathbf{I}$ . However, a number of empirical studies (e.g., Cochrane, 1994; Cogley, 2005) argue that incorporating information about consumption is informative about the permanent component in GDP as predicted by the permanent income hypothesis. The theory predicts that consumers, smoothing consumption throughout their lifetime, should react more strongly to permanent, as opposed to transitory, changes in income. As a consequence, looking at GDP and consumption data together will help in separating growth into long-run and cyclical fluctuations.<sup>7</sup> Therefore, our baseline specification imposes that consumption and output grow at the same rate  $g_t$  in the long run. On the contrary, we do not impose that investment also grows at this rate, as would be the case in the basic neoclassical growth model, since the presence of investment-specific technological change implies that real investment has a different low-frequency trend (Greenwood, Hercowitz, & Krussel, 1997).

Formally, ordering real GDP and consumption growth first, and setting  $m = 2$  and  $r = 1$ , this is represented as

$$\mathbf{a}_t = g_t, \quad \mathbf{B} = [1 \ 1]'. \quad (8)$$

Note that in this baseline specification, we model time variation only in the intercept for GDP and consumption while leaving it constant for the other observables. Of course, it may be the case that some of the remaining  $n - m$  series in  $\mathbf{y}_t$  feature low-frequency variation in their means. For instance, this could be the case for investment. The key question is whether leaving it unspecified will affect the estimate of the long-run growth rate of GDP, our main object

<sup>5</sup> For the purpose of tracking real GDP with a large number of closely related activity indicators, the use of one factor is appropriate. Also note that we order real GDP growth as the first element of  $\mathbf{y}_t$  and normalize the loading for GDP to unity. This serves as an identifying restriction in our estimation algorithm. Bai and Wang (2015) discuss minimal identifying assumptions for DFMs.

<sup>6</sup> We demonstrate this point with the use of Monte Carlo simulations, showing that a random walk trend in real GDP growth learns quickly about a discrete break once it has occurred. However, the random walk does not detect a drift when there is not one, despite the presence of a large cyclical component. Online appendix C provides a discussion and the full results of these simulations.

<sup>7</sup> While a strict interpretation of the permanent income hypothesis is rejected in the data, from an econometric point of view, the statement applies as long as permanent changes are the main driver of consumption. See Cochrane (1994) for a very similar discussion.

of interest. We ensure that this is not the case by allowing for persistence (and, in particular, we do not rule out unit roots) in the idiosyncratic components. If a series does feature a unit root that is not included in  $\mathbf{a}_t$ , its trend component will be absorbed by the idiosyncratic component. The choice of which elements to include in  $\mathbf{a}_t$  therefore reflects the focus of a particular application.<sup>8</sup> Of course, if two series share the same underlying low-frequency component and this is known with certainty, explicitly accounting for the shared low-frequency variation will improve the precision of the estimation, but the risk of incorrectly including the trend is much larger than the risk of incorrectly excluding it. Therefore, in our baseline specification we include in  $\mathbf{a}_t$  the intercept for GDP and consumption, while leaving any possible low-frequency variation in other series to be captured by the respective idiosyncratic components.<sup>9</sup>

An extension to include additional time-varying intercepts is straightforward through the flexible construction of  $\mathbf{c}_t$  in equation (2). In fact, in section V, we explore how interest in the low-frequency movements of additional series leads to alternative choices for  $\mathbf{a}_t$  and  $\mathbf{B}$ .<sup>10</sup>

### C. Dealing with Mixed Frequencies and Missing Data

Tracking activity in real time requires a model that can efficiently incorporate information from series measured at different frequencies. In particular, it must include both quarterly variables, such as the growth rate of real GDP, as well as more timely monthly indicators of real activity. Therefore, the model is specified at monthly frequency, and following Mariano and Murasawa (2003), the (observed) quarterly growth rates of a generic quarterly variable,  $x_t^q$ , can be related to the (unobserved) monthly growth rate  $x_t^m$  and its lags using a weighted mean. Specifically,

$$x_t^q = \frac{1}{3}x_t^m + \frac{2}{3}x_{t-1}^m + x_{t-2}^m + \frac{2}{3}x_{t-3}^m + \frac{1}{3}x_{t-4}^m, \quad (9)$$

<sup>8</sup> In principle, these unmodeled trends could still be recovered from our specification by applying a Beveridge-Nelson decomposition to its estimated idiosyncratic component. In practice, any low-frequency variation in the idiosyncratic component is likely to be obscured by a large amount of high-frequency noise in the data, and as result, the extracted Beveridge-Nelson trend component will be imprecisely estimated and will not be smooth. In our specification, the elements of  $\mathbf{a}_t$  are instead extracted directly, so that we are able to improve the extraction by imposing additional assumptions (e.g., smoothness) and prior beliefs (e.g., low variability) on its properties.

<sup>9</sup> We confirm this line of reasoning with a series of Monte Carlo experiments in which data are generated from a system that features low-frequency movements in more series, which are left unmodeled in the estimation. In both the case of series with independent trends and series that share the trend of interest, the fact that they are left unmodeled has little impact on the estimate of the latter. Online appendix C presents further discussion and the full results of these simulations.

<sup>10</sup> Note that the limiting case explicitly models time-varying intercept in all indicators, so that  $m = r = n$  and  $\mathbf{B} = \mathbf{I}_n$ , that is, an identity matrix of dimension  $n$ . See Creal, Koopman, and Zivot (2010) and Fleischman and Roberts (2011) for similar approaches. This setup would imply that the number of state variables increases with the number of observables, which severely increases the computational burden of the estimation while offering little additional evidence with respect to the focus of this paper.

and only every third observation of  $x_t^q$  is actually observed. Substituting the corresponding line of equation (1) into equation (9) yields a representation in which the quarterly variable depends on the factor and its lags. The presence of mixed frequencies is thus reduced to a problem of missing data in a monthly model.

Besides mixed frequencies, additional sources of missing data in the panel include the “ragged edge” at the end of the sample, which stems from the nonsynchronicity of data releases; missing data at the beginning of the sample, since some data series have been created or collected more recently than others; and missing observations due to outliers and data collection errors. Our Bayesian estimation method exploits the state-space representation of the DFM and jointly estimates the latent factors, the parameters, and the missing data points using the Kalman filter.

### D. State-Space Representation and Estimation

The model features autocorrelated idiosyncratic components (see equation [4]). In order to cast it in state-space form, we include the idiosyncratic components of the quarterly variables in the state vector, and we redefine the system for the monthly indicators in terms of quasi-differences (see, e.g., Kim & Nelson, 1999b).<sup>11</sup> The model is estimated with Bayesian methods simulating the posterior distribution of parameters and factors using a Markov chain Monte Carlo (MCMC) algorithm. We closely follow the Gibbs-sampling algorithm for DFMs proposed by Bai and Wang (2015) but extend it to include mixed frequencies, the time-varying intercept, and SV. The SVs are sampled using the approximation of Kim, Shephard, and Chib (1998), which is considerably faster than the exact Metropolis-Hastings algorithm of Jacquier, Polson, and Rossi (2002). Our complete sampling algorithm, together with the details of the state-space representation, can be found in section D of the online appendix.

## IV. Results for U.S. Data

### A. Data Selection

Our data set includes four key business cycle variables measured at quarterly frequency (output, consumption, investment, and aggregate hours worked), as well as a set of 24 monthly indicators that are intended to provide additional information about cyclical developments in a timely manner.

The included quarterly variables are strongly procyclical and are considered key indicators of the business cycle

<sup>11</sup> Since the quarterly variables are observed only every third month, we cannot take the quasi-difference for their idiosyncratic components, which are instead added as an additional state with the corresponding transition dynamics. Banbura and Modugno (2014) suggest including all of the idiosyncratic components as additional elements of the state vector. Our solution has the desirable feature that the number of state variables will increase with the number of quarterly variables rather than the total number of variables leading to a gain of computational efficiency.

(see, e.g., Stock & Watson, 1999). Furthermore, theory predicts that they will be useful in disentangling low-frequency movements from cyclical fluctuations in output growth. Indeed, as discussed in section IIIB, the permanent income hypothesis predicts that consumption data will be particularly useful for the estimation of the long-run growth component,  $g_t$ .<sup>12</sup> However, investment and hours worked are very sensitive to cyclical fluctuations and thus will be particularly informative for the estimation of the common factor,  $f_t$ .<sup>13</sup>

The additional monthly indicators are crucial to our objective of disentangling in real time the cyclical and long-run components of GDP growth, since the quarterly variables are only available with substantial delay. In principle, a large number of candidate series are available to inform the estimate of  $f_t$ , and indirectly, of  $g_t$ . In practice, however, macroeconomic data series are typically clustered in a small number of broad categories (such as production, employment, or income) for which disaggregated series are available along various dimensions (such as economic sectors, demographic characteristics, or expenditure categories). The choice of which available series to include for estimation can therefore be broken into, first, a choice of which broad categories to include, and second, to which level and along which dimensions of disaggregation.

With regard to which broad categories of data to include, previous studies agree that prices and monetary and financial indicators are uninformative for the purpose of tracking real GDP, and argue for extracting a single common factor that captures real economic activity.<sup>14</sup> As for the possible inclusion of disaggregated series within each category, Boivin and Ng (2006) argue that the presence of strong correlation in the idiosyncratic components of disaggregated series of the same category will be a source of misspecification that can worsen the performance of the model in terms of in-sample fit and out-of-sample forecasting of key series.<sup>15</sup> Alvarez et al. (2012) investigate the trade-off between DFMs with very few indicators, where the good large-sample properties of factor models are unlikely to hold, and those with a very large amount of indicators, where the problems above

are likely to arise. They conclude that using a medium-sized panel with representative indicators of each category yields the best forecasting results.

The above considerations lead us to select 24 monthly indicators that include the high-level aggregates for all of the available broad categories that capture real activity, without overweighting any particular category. The complete list of variables contained in our data set is presented in table 1. As the table shows, we include representative series of expenditure and income, the labor market, production and sales, foreign trade, housing, and business and consumer confidence.<sup>16</sup> The inclusion of all the available monthly surveys is particularly important. Apart from being the most timely series available, these are unlikely to feature permanent shifts in their mean by construction and have a high signal-to-noise ratio. They thus provide a clean signal to separate the cyclical component of GDP growth from its long-run counterpart. In section IVF, we explore the sensitivity of our results to the size and composition of the data panel used.

Our panel spans the period January 1947 to March 2015. The start of our sample coincides with the year for which quarterly national accounts data are available from the Bureau of Economic Analysis. This enables us to study the evolution of long-run growth over the entire postwar period.<sup>17</sup>

## B. Model Settings and Priors

The choice of the data set justifies the single-factor structure of the model.  $f_t$  in this case can be interpreted as a coincident indicator of real economic activity (see, e.g., Stock & Watson, 1989, and Mariano & Murasawa, 2003). The number of lags in the polynomials  $\phi(L)$  and  $\rho(L)$  is set to  $p = 2$  and  $q = 2$  as in Stock and Watson (1989). We wish to impose as little prior information as possible, so we use uninformative priors for the factor loadings and the autoregressive coefficients of factors and idiosyncratic components. The variances of the innovations to the time-varying parameters,  $\omega_a^2$ ,  $\omega_e^2$ , and  $\omega_{\eta,i}^2$  in equations (5) to (7), are, however, difficult to identify from the information contained in the likelihood alone. As the literature on Bayesian VARs documents, attempts to use noninformative priors for these parameters will in many cases produce posterior estimates, which imply a relatively large amount of time variation. While this will tend to improve the in-sample fit

<sup>12</sup> Due to the presence of faster technological change in the durable goods sector, there is a downward trend in the relative price of durable goods. As a consequence, measured consumption grows faster than overall GDP. Following a long tradition in the literature (see, e.g., Whelan, 2003), we construct a Fisher index of nondurables and services and use its growth rate as an observable variable in the panel. It can be verified that the ratio of consumption defined in this manner to real GDP displays no trend in the data, unlike the trend observed in the ratio of overall consumption to GDP.

<sup>13</sup> We define investment as a chain-linked aggregate of business fixed investment and consumption of durable goods, which is consistent with our treatment of consumption. In order to obtain a measure of hours for the total economy we follow the methodology of Ohanian and Raffo (2012).

<sup>14</sup> Giannone et al. (2005) conclude that that prices and monetary indicators do not contribute to the precision of GDP nowcasts. Forni et al. (2003) and Stock and Watson (2003) find at best mixed results for financial variables.

<sup>15</sup> This problem is exacerbated by the fact that more detailed disaggregation levels and dimensions are available for certain categories of data, such as employment, meaning that the disaggregation will automatically tilt the factor estimates toward that category.

<sup>16</sup> When there are multiple candidates for the high-level aggregate of a category, we include both. For example, we include employment as measured by both the establishment and household surveys, and consumer confidence as surveyed by both the Conference Board and the University of Michigan.

<sup>17</sup> We take full advantage of the Kalman filter's ability to deal with missing observations at any point in the sample, and we are able to incorporate series that become available substantially later than 1947, up to as late as 2007. Note that for consumption expenditures, monthly data became available in 1959, whereas quarterly data are available from 1947. In order to use all available data, we apply the polynomial in equation (9) to the monthly data and treat the series as quarterly, with available observations for the last month of the quarter for 1947 to 1958 and for all months since 1959.



TABLE 1.—DATA SERIES USED IN EMPIRICAL ANALYSIS

	Type	Start Date	Transform.	Lag
Quarterly time series				
Real GDP	Expenditure and income	Q2:1947	% QoQ Ann	26
Real consumption (excl. durables)	Expenditure and income	Q2:1947	% QoQ Ann	26
Real investment (incl. durable consumption)	Expenditure and income	Q2:1947	% QoQ Ann	26
Total hours worked	Labor Market	Q2:1948	% QoQ Ann	28
Monthly indicators				
Real personal income less transfers	Expenditure and income	Feb 59	% MoM	27
Industrial production	Production and sales	Jan 47	% MoM	15
New orders of capital goods	Production and sales	Mar 68	% MoM	25
Real retail sales and food services	Production and sales	Feb 47	% MoM	15
Lightweight vehicle sales	Production and sales	Feb 67	% MoM	1
Real exports of goods	Foreign trade	Feb 68	% MoM	35
Real imports of goods	Foreign trade	Feb 69	% MoM	35
Building permits	Housing	Feb 60	% MoM	19
Housing starts	Housing	Feb 59	% MoM	26
New home sales	Housing	Feb 63	% MoM	26
Payroll empl. (Establishment Survey)	Labor market	Jan 47	% MoM	5
Civilian empl. (Household Survey)	Labor market	Feb 48	% MoM	5
Unemployed	Labor market	Feb 48	% MoM	5
Initial claims for unemployment insurance	Labor market	Feb 48	% MoM	4
Monthly indicators (soft)				
Markit Manufacturing PMI	Business confidence	May 07	-	-7
ISM manufacturing PMI	Business confidence	Jan 48	-	1
ISM nonmanufacturing PMI	Business confidence	Jul 97	-	3
NFIB Small Business Optimism Index	Business confidence	Oct 75	Diff 12 M.	15
U. of Michigan: Consumer Sentiment	Consumer confidence	May 60	Diff 12 M.	-15
Conference Board: Consumer Confidence	Consumer confidence	Feb 68	Diff 12 M.	-5
Empire State Manufacturing Survey	Business (regional)	Jul 01	-	-15
Richmond Fed Mfg Survey	Business (regional)	Nov 93	-	-5
Chicago PMI	Business (regional)	Feb 67	-	0
Philadelphia Fed Business Outlook	Business (regional)	May 68	-	0

% QoQ Ann refers to the quarter-on-quarter annualized growth rate, % MoM refers to  $(y_t - y_{t-1})/y_{t-1}$ , and Diff 12 M. refers to  $y_t - y_{t-12}$ . The last column shows the average publication lag: the number of days elapsed from the end of the period that the data point refers to until its publication by the statistical agency. All series were obtained from the Haver Analytics database.

of the model, it is also likely to worsen out-of-sample forecast performance. We therefore use priors to shrink these variances toward 0—toward the standard DFM that excludes time-varying long-run GDP growth and SV. In particular, for  $\omega_a^2$ , we set an inverse gamma prior with 1 degree of freedom and scale equal to 0.001.<sup>18</sup> For  $\omega_\epsilon^2$  and  $\omega_{\eta,i}^2$ , we set an inverse gamma prior with 1 degree of freedom and scale equal to 0.0001, closely following Cogley and Sargent (2005) and Primiceri (2005).<sup>19</sup> We estimate the model with 7,000 replications of the Gibbs-sampling algorithm, of which the first 2,000 are discarded as burn-in draws and the remaining ones are kept for inference.

### C. In-Sample Results

Figure 2a plots the posterior median, together with the 68% and 90% posterior credible intervals of the long-run growth rate of real GDP. This estimate is conditional on the entire sample and accounts for both filtering and parameter uncertainty. Several features of our estimate of long-run growth are worth noting. While the growth rate is stable

between 3% and 4% during the first decades of the post-war period, a slowdown is clearly visible from around the late 1960s through the 1970s, consistent with the “productivity slowdown” (Nordhaus, 2004). The acceleration of the late 1990s and early 2000s associated with the productivity boom in the IT sector (Oliner & Sichel, 2000) is also visible. Thus, until the middle of the first decade of the 2000s, our estimate conforms well to the generally accepted narrative about fluctuations in potential growth.<sup>20</sup> More recently, after peaking at about 3.5% in 2000, the median estimate of the long-run growth rate has fallen to about 2% in early 2015, a more substantial decline than the one observed after the productivity slowdown of the 1970s. Moreover, the slowdown appears to have happened gradually since the start of the 2000s, with most of the decline having occurred before the Great Recession.<sup>21</sup> Interestingly, a small rebound is visible at the end of the sample, but long-run growth stands far

<sup>20</sup> Online appendix G provides a comparison of our estimate with the Congressional Budget Office (CBO) measure of potential growth, with some additional discussion.

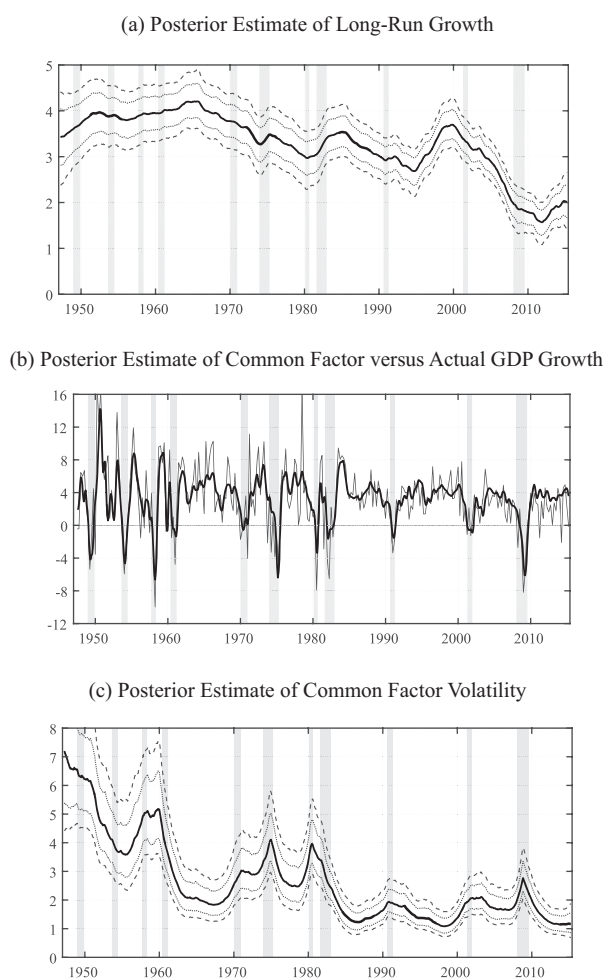
<sup>21</sup> In principle, it is possible that our choice of modeling long-run GDP growth as a random walk is hard-wiring into our conclusion that the decline happened in a gradual way. In experiments with simulated data, presented in section C of the online appendix, we show that if changes in long-run growth occur in the form of discrete breaks rather than evolving gradually, the (one-sided) filtered estimates will exhibit a discrete jump at the moment of the break. Instead, for U.S. data, the filtered estimates of the long-run growth component also decline in a gradual manner (see figure A.1 in online appendix A).

<sup>18</sup> To gain an intuition about this prior, note that over a period of ten years, this would imply that the random walk process of the long-run growth rate is expected to vary with a standard deviation of around 0.4 percentage points in annualized terms, a fairly conservative prior.

<sup>19</sup> We provide further explanations and address robustness to the choice of priors in online appendix F.



FIGURE 2.—TREND, CYCLE, AND VOLATILITY, 1947–2015 (% ANNUAL GROWTH RATE)



(a) The posterior median (solid line), together with the 68% and 90% (dotted and dashed lines) posterior credible intervals of long-run real GDP growth. (b) Actual real GDP growth (thin line) against the posterior median estimate of the common factor, aligned with the mean of real GDP growth (thick line). (c) The median (solid line) and the 68% and the 90% (dotted and dashed lines) posterior credible intervals of the volatility of the common factor, that is, the square root of  $\text{var}(f_t) = \sigma_{\epsilon,t}^2(1 - \phi_2)/[(1 + \phi_2)((1 - \phi_2)^2 - \phi_1^2)]$ . Shaded areas represent NBER recessions.

below its postwar average of 3.2%, with the 90% posterior credible interval ranging from 1.5% to 2.5%.

Figure 2b plots the time series of quarterly real GDP growth, together with the median posterior estimates of the common factor, aligned with the mean of real GDP growth. This plot highlights how the common factor captures the bulk of business cycle frequency variation in output growth, while higher-frequency, quarter-to-quarter variation is attributed to the idiosyncratic component. In the latter part of the sample, GDP growth is visibly below the factor, reflecting the decline in long-run growth.

The posterior estimate of the SV of the common factor is presented in figure 2c. It is clearly visible that volatility declines over the sample. The late 1940s and 1950s were extremely volatile, with a first large drop in volatility in the early 1960s. The Great Moderation is also clearly visible, with the average volatility pre-1985 being much larger than

the average of the post-1985 sample. Notwithstanding the large increase in volatility during the Great Recession, our estimate of the common factor volatility since then remains consistent with the Great Moderation still being in place. This confirms the early evidence reported by Gadea-Rivas, Gomez-Loscos, and Perez-Quiros (2014). It is clear from the figure that volatility spikes during recessions, a feature that brings our estimates close to the recent findings of Jurado et al. (2015) relating to business cycle uncertainty.<sup>22</sup> It appears that the random walk specification is flexible enough to capture cyclical changes in volatility as well as permanent phenomena such as the Great Moderation. Online appendix A contains analogous charts for the volatilities of the idiosyncratic components of selected data series. Similar to the volatility of the common factor, many of the idiosyncratic volatilities present sharp increases during recessions.

The results provide evidence that a significant decline in long-run U.S. real GDP growth occurred over the past decade and are consistent with a relatively gradual decline since the early 2000s. Our estimates show that the bulk of the slowdown from the elevated levels of growth at the turn of the century occurred before the Great Recession, which is consistent with the narrative of Fernald (2014) on the fading of the IT productivity boom. This recent decline is the largest movement in long-run growth observed in the postwar period.

#### D. Real-Time Results

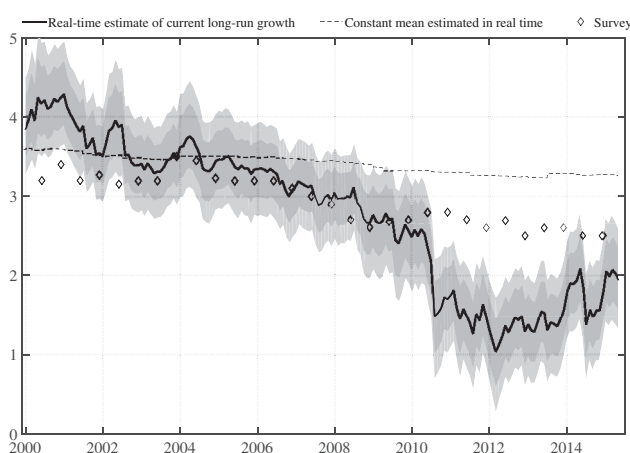
As Orphanides (2003) emphasized, macroeconomic time series are heavily revised over time, and in many cases these revisions contain valuable information that was not available at initial release. Therefore, it is important to assess, using the data available at each point in time, when the model detected the slowdown in long-run growth. For this purpose, we reconstruct our data set using vintages of data available from the Federal Reserve Bank of St. Louis ALFRED database. Our aim is to replicate as closely as possible the situation of a decision maker who would have applied our model in real time. We fix the start of our sample in 1947:Q1 and use an expanding out-of-sample window that starts on January 11, 2000, and ends on June 30, 2015. This is the longest possible window for which we are able to include the entire panel in table 1 using fully real-time data. We then proceed by reestimating the model each day in which new data are released.<sup>23</sup>

<sup>22</sup> It is interesting to note that while in our model the innovations to the level of the common factor and its volatility are uncorrelated, the fact that increases in volatility are observed during recessions indicates the presence of negative correlation between the first and second moments, implying negative skewness in the distribution of the common factor. We believe a more explicit model of this feature is an important priority for future research.

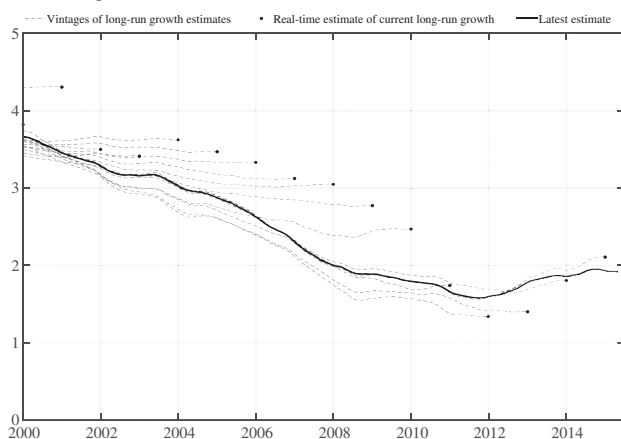
<sup>23</sup> In a few cases, new indicators were developed after January 2000. For example, the Markit Manufacturing PMI survey is currently one of the most timely and widely followed indicators, but it started being conducted in 2007. In those cases, we append to the panel, in real time, the vintages

FIGURE 3.—LONG-RUN GDP GROWTH ESTIMATES IN REAL TIME

## (a) Evolution of the Current Assessment of Long-Run Growth



## (b) Selected Vintages of Long-Run Growth Estimates Using Real-Time Data



The figure presents results from reestimating the model using the vintage of data available at each point in time from January 2000 to March 2015. The start of the estimation sample is fixed at Q1:1947. (a) The median real-time estimate of current long-run growth over time. This is the locus traced by the end points of all vintages. The shaded areas around the solid line represent the 68th and 90th percentiles. The dashed line is the contemporaneous estimate of the historical average of real GDP growth. The diamonds are the median response to the Philadelphia Fed Livingston Survey of Professional Forecasters on the average growth rate for the next 10 years. (b) The median estimate of long-run GDP growth for various vintages of data (dashed gray lines). The estimate of the latest vintage is shown as the solid thick line. Gray shaded areas represent NBER recessions in both panels.

Figure 3 looks at the model's real-time assessment of long-run growth at various points in time. Figure 3a plots the real-time estimate of current long-run growth, with 68% and 90% uncertainty bands. For comparison, the panel also shows the median response to the Philadelphia Fed Livingston Survey of Professional Forecasters (SPF) on the average growth rate for the next ten years and the estimate of long-run growth from a model with a constant intercept for GDP growth. The latter estimate is also updated as new information arrives but weighs all points of the

of the new indicators as soon as sufficient history is available. In the example of the PMI, this is the case since mid-2012. By implication, the number of indicators in our data panel grows when new indicators appear. Full details about the construction of the vintage database are available in online appendix E.

sample equally. Figure 3b displays vintages of the median long-run growth estimate using information available up to July of each year. The locus traced by the end point of each vintage corresponds to the current real-time estimate of figure 3a.

The evolution of the baseline model's estimate of long-run growth when estimated in real time declines gradually from a peak of about 4% in early 2000 to around 2.5% just after the end of the Great Recession. From this time, the constant estimate shown in figure 3a is always outside the 90% posterior bands. There is a sharp reassessment of long-run growth around July 2010, coinciding with the publication by the Bureau of Economic Analysis (BEA) of the annual revisions to the National Accounts, which each year incorporate previously unavailable information for the previous three years. The revisions implied a substantial downgrade, in particular, to the growth of consumption in the first year of the recovery, from 2.5% to 1.6%, and instead allocated much of the growth in GDP during the recovery to inventory accumulation.<sup>24</sup> Reflecting the role of consumption as the most persistent and forward-looking component of GDP, the estimate of long-run growth is downgraded sharply. Figure 3b shows how the 2010 revisions in fact trigger a reinterpretation of the years leading to the Great Recession. With the revised information, the bulk of the slowdown in long-run growth is now estimated to have occurred before the recession.<sup>25</sup> From 2010 onward, the model predicts a recovery that is extremely slow by historical standards. This is four years before the structural break test detected a statistically significant decline.<sup>26</sup> It is evident from the preceding discussion that revisions to past data by the BEA are an important source of changes to the long-run growth estimate in real time. Since the revision process is not modeled explicitly within the DFM, the in-sample results of section IVC do not take into account the uncertainty stemming from future revisions. Interestingly, in the latest part of the sample, the estimate of long-run growth has recovered slightly to about 2%, but this has been triggered by improvements in incoming data rather than revisions to past vintages.

With regard to the SPF, it is noticeable that from 2003 to about 2010, the survey is remarkably similar to the model, but since then, the SPF forecast has continued to drift down only very slowly, standing at 2.5% as of mid-2015. It is noteworthy that as Stanley Fischer pointed out in the speech quoted at the start of the paper, during that period, both private and institutional forecasters systematically overestimated growth.

<sup>24</sup> See online appendix I for additional figures on the National Accounts revisions during this period.

<sup>25</sup> Indeed, the (one-sided) filtered estimate based on the latest vintage, which ignores the effect of data revisions, displays a more gradual pattern of decline (see figure A.1 in section A of the online appendix).

<sup>26</sup> A simpler specification that does not use consumption to inform the trend would detect the decline in long-run growth one year later, with additional revisions to past GDP in July 2011.

### E. *Implications for Nowcasting GDP*

The standard DFM with constant long-run growth and constant volatility has been successfully applied to produce current quarter nowcasts of GDP (see Banbura et al., 2010, for a survey). Using our real-time U.S. database, we carefully evaluate whether the introduction of time-varying long-run growth and SV into the DFM framework also improves the performance of the model along this dimension. We find that over the evaluation window 2000 to 2015, the model is at least as accurate at point forecasting and significantly better at density forecasting than the benchmark DFM. Most of the improvement in density forecasting comes from correctly assessing the center and the right tail of the distribution, implying that the time-invariant DFM is assigning excessive probability to a strong recovery. In an evaluation subsample spanning the postrecession period, the relative performance of both point and density forecasts improves substantially, coinciding with the significant downward revision of the model's assessment of long-run growth. In fact, ignoring the variation in long-run GDP growth would have resulted in being on average around 1 percentage point too optimistic from 2009 to 2015.<sup>27</sup>

To sum up, the addition of the time-varying components not only provides a tool for decision makers to update their knowledge about the state of long-run growth in real time. It also brings about a substantial improvement in short-run forecasting performance when the trend is shifting, without worsening the forecasts when the latter is relatively stable. The proposed model therefore provides a robust and timely methodology to track GDP when long-run growth is uncertain.

### F. *Inspecting the Role of Data Set Size and Composition*

The DFM exploits the cross-sectional dimension, and not just the time series dimension, in separating cycle from trend. It is interesting to quantify the advantage that the DFM provides over traditional trend cycle decompositions and to investigate the robustness of our main conclusions to alternative data sets of varying size and composition. In order to do so, we consider (a) a bivariate model with GDP and unemployment only (labeled “Okun”), (b) an intermediate model with GDP and the four additional variables often included in the construction of coincident indicators (see Mariano & Murasawa, 2003, and Stock & Watson, 1989) (labeled “MM03”), (c) our “Baseline” specification with 28 variables, and (d) an “Extended” model that uses disaggregated data for many of the headline series included in the baseline specification, totaling 155 variables.<sup>28</sup> Moreover,

<sup>27</sup> Online appendix H provides the full details of the forecast evaluation exercise.

<sup>28</sup> As we argue in section IVA, the introduction of a large number of disaggregated series, even if related to real activity, is likely to lead to model misspecification whenever the sectoral data are not contemporaneously related. For the extended specification, we consider a solution to this problem that allows maintaining the parsimonious one-factor structure.

in order to investigate the gains associated with imposing additional structure to long-run GDP growth, for the last two specifications, we also consider a version of the model that does not impose common long-run growth in GDP and consumption.

The top panel of table 2 reports the mean point estimates for each specification over selected subsamples.<sup>29</sup> In all cases, the results are consistent with a decline in the long-run growth rate in the last part of the sample. Quantitatively, most specifications are very close to the baseline, with the specifications that impose common long-run growth in GDP and consumption finding an earlier and sharper decline. The exception is the Okun specification, which instead estimates a smaller increase in the mid-1990s, as well as a larger decline in long-run growth in the past decade. It is noteworthy that the mean estimate of the extended specification is very close to that of the baseline.

The lower portion of table 2 instead investigates the uncertainty around the mean estimates. The uncertainty around the long-run growth estimate declines as we move from the bivariate to the multivariate specifications, with most of the reduction happening once a handful of variables are included. When the panel is extended to include a large number of disaggregated series, the uncertainty increases.<sup>30</sup> While including a few key series, such as the ones in the specification of Mariano and Murasawa (2003), seems to already achieve the bulk of the reduction in uncertainty, it should be taken into account that those variables are available only with a relatively long publication lag and subject to considerable revisions over time. Our proposed strategy of using an intermediate number of indicators, including the more timely and accurate surveys, is likely to lead to more satisfactory results in a real-time setting. Furthermore, the inclusion of the surveys is helpful in identifying the long-run growth rate, as those variables do not display a time-varying long-run mean by construction.

Overall this exercise highlights that the finding of a substantial decline in the long-run growth rate is confirmed across different specifications that use data sets of varying size and composition. The baseline specification, which uses an intermediate number of series including both hard data and surveys, leads to the lowest uncertainty around the

By extending the model to include lags of the factor in the observation equation, each variable can display heterogeneous responses to the common factor, and correlation between idiosyncratic components is reduced. Given that the extended model is heavily parameterized, we follow D'Agostino et al. (2015) in choosing priors that shrink the model toward the contemporaneous-only specification, which is nested in the extended case. Full details and the composition of the data set and the changes to the estimation in case of the extended model are provided in online appendix J.

<sup>29</sup> See online appendix J for a comparison of the results of each alternative specification with the baseline results over the entire sample.

<sup>30</sup> We conjecture that as many more variables are added, the fit of the common factor to the cyclical component of GDP worsens. As a consequence, some cyclical variation of GDP spills over to the estimate of the long-run component. The uncertainty around the common factor, on the other hand, continues to decline.

TABLE 2.—COMPARISON OF RESULTS FOR ALTERNATIVE DATA SETS AND SPECIFICATIONS

	<i>Okun</i>	<i>MM03</i>	Baseline		Extended	
			<i>GDP Only</i>	<i>GDP + C</i>	<i>GDP Only</i>	<i>GDP + C</i>
<i>Long-run growth</i>						
1947–1972	3.9	3.5	3.6	<b>3.8</b>	3.6	3.9
1973–1995	3.2	3.4	3.1	<b>3.1</b>	3.2	3.2
1996–2007	2.6	3.2	3.1	<b>3.1</b>	3.0	3.1
2008–2015	1.6	2.5	2.4	<b>1.8</b>	2.2	1.7
End of sample	1.3	2.4	2.3	<b>2.0</b>	2.1	2.0
<i>Uncertainty: Long run</i>						
Filtered	0.82	0.63	0.64	<b>0.56</b>	0.78	0.63
Smoothed	0.44	0.36	0.37	<b>0.35</b>	0.44	0.39
<i>Uncertainty: Cycle</i>						
Filtered	2.08	1.47	0.79	<b>0.76</b>	0.23	0.23
Smoothed	1.89	1.32	0.62	<b>0.60</b>	0.25	0.25

Each column presents the estimation results corresponding to the alternative models (data sets) considered in this section. The table displays the posterior means of the upper growth rate of real GDP over selected subsamples and the posterior uncertainty corresponding to both the long-run growth rate of real GDP, as well as the common factor. The uncertainty is calculated as an average over the entire sample. The column in bold highlights the baseline specification used throughout Section IV.

long-run growth estimate. Our results have important implications for trend-cycle decompositions of output, which usually include only a few cyclical indicators, generally inflation or variables that are direct inputs to the production function (see, e.g., Gordon, 2014a, or Reifschneider et al., 2013). As we show, greater precision of the trend component can be achieved by exploiting the common cyclical features of additional macroeconomic variables.

## V. Decomposing Movements in Long-Run Growth

In this section, we show how our model can be used to decompose the long-run growth rate of output into long-run movements in labor productivity and labor input. By doing this, we exploit the ability of the model to filter away cyclical variation and idiosyncratic noise and obtain clean estimates of underlying long-run trends. We see this exercise as a step toward giving an economically more meaningful interpretation to the movements in long-run real GDP growth detected by our model.

GDP growth is by identity the sum of growth in output per hour and growth in total hours worked. It is therefore possible to split the long-run growth trend in our model into two orthogonal components such that this identity is satisfied in the long run. Here we make use of our flexible definition of  $\mathbf{c}_t$  in equation (2). In particular, ordering the growth rates of real GDP, real consumption, and total hours as the first three variables in  $\mathbf{y}_t$ , we define

$$\mathbf{a}_t = \begin{bmatrix} z_t \\ h_t \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad (10)$$

so that the model is specified with two time-varying components, the first of which loads output and consumption but not hours and the second loads all three series. The first component is then by construction the long-run growth rate of labor productivity, while the second one captures low-frequency

movements in labor input independent of productivity.<sup>31</sup> Given the relation in equation (10), the two components add up to the time-varying intercept in the baseline specification:  $g_t = z_t + h_t$ .<sup>32</sup> It follows from standard growth theory that our estimate of the long-run growth rate of labor productivity will capture both technological factors and other factors, such as capital deepening and labor quality.<sup>33</sup>

Figure 4 presents the results of the decomposition exercise for the United States. Figure 4a plots the median posterior estimate of long-run real GDP growth and its labor productivity and total hours components. The posterior bands for long-run real GDP growth are included. The time series evolution conforms very closely to the narrative of Fernald (2014), with a pronounced boom in labor productivity in the mid-1990s and a subsequent fall in the 2000s clearly visible. The decline in the 2000s is relatively sudden, while the 1970s slowdown appears as a more gradual phenomenon starting in the late 1960s. Furthermore, the results reveal that during the 1970s and 1980s, the impact of the productivity slowdown on output growth was partly masked by a secular increase in hours, probably reflecting increases in the working-age population as well as labor force participation (see, e.g., Goldin, 2006). Focusing on the period since 2000, labor productivity accounts for almost the entire decline.<sup>34</sup> This contrasts

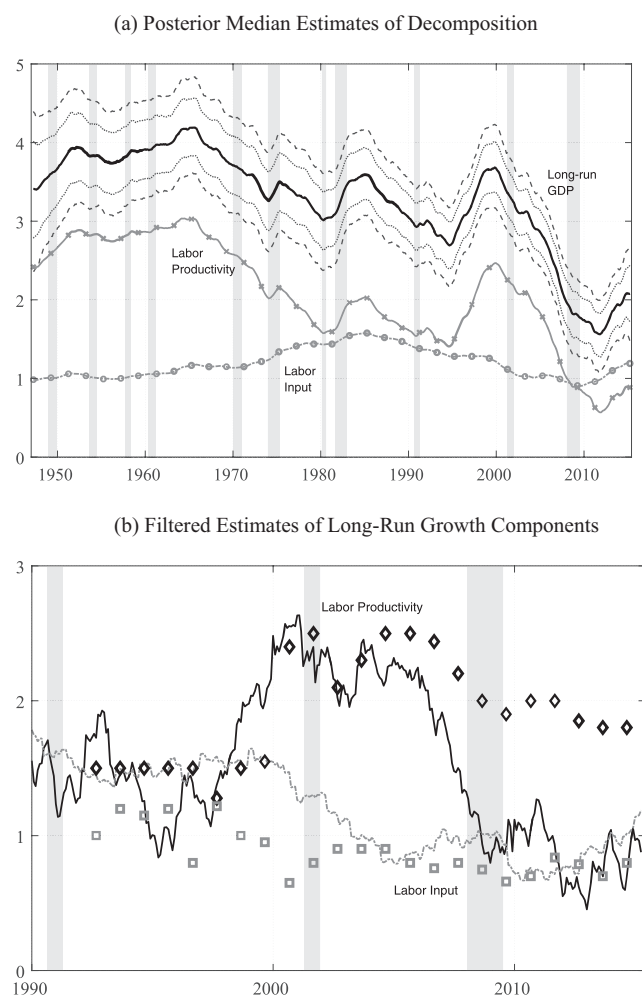
<sup>31</sup>  $z_t$  and  $h_t$  jointly follow random walks with diagonal covariance matrix as defined by equation (7). Restricting the covariance matrix is not necessary for estimation, but imposing it allows us to interpret the innovations to the trends as exogenous shocks to the long-run growth rates of the variables. The hours trend is therefore interpreted as those low-frequency movements in hours that are uncorrelated with labor productivity. Allowing for a full covariance matrix would yield trends that are linear combinations of the current ones, but would lack a clear economic interpretation.

<sup>32</sup> Since  $z_t$  and  $h_t$  are independent and add up to  $g_t$ , we set the prior on the scale of their variances to half of the one set in section IVB on  $g_t$ .

<sup>33</sup> Further decomposing  $z_t$  into technology and non-technology movements requires additional information to separately identify these components. One possibility, which we explore in online appendix K, is to use an independent measure of TFP to isolate technological factors. Note, however, that reliable data on capital input, labor quality, or estimates of TFP are not available in real time, making the focus on long-run labor productivity more appealing in a real-time setting.

<sup>34</sup> In online appendix K, we extend the analysis to decompose the labor productivity trend into long-run TFP and nontechnological forces. We find that TFP accounts for virtually all of the slowdown.

FIGURE 4.—DECOMPOSITION OF LONG-RUN U.S. OUTPUT GROWTH



(a) The posterior median (solid black line), together with the 68% and 90% (dotted and dashed gray lines) posterior credible intervals of long-run GDP growth and the posterior median of both long-run labor productivity growth and long-run total hours growth (crossed markers and circled markers). (b) The filtered estimates of the two components  $\hat{z}_{it}$  and  $\hat{h}_{it}$  since 1990. For comparison, the corresponding forecasts from the SPF are plotted (diamonds and squares). The SPF forecast for total hours is obtained as the difference between the forecasts for real GDP and labor productivity.

explanations by which slow labor force growth has been a drag on GDP growth. When taking away the cyclical component of hours and focusing solely on its long-run component, the contribution of hours has, if anything, accelerated since the Great Recession. Figure 4b presents the filtered estimates of the two components, that is, the output of the Kalman filter which uses data only up to each point in time. For comparison, the corresponding SPF forecasts are included. Most notably, this plot reveals that starting around 2005, a relatively sharp revision to labor productivity drives the decline in long-run output growth.<sup>35</sup> Interestingly, the professional forecasters have been very slow in incorporating the productivity slowdown into their long-run forecasts. This

<sup>35</sup> In an additional figure, provided in section A of the online appendix, we plot 5,000 draws from the joint posterior distribution of the variances of the innovations to the labor productivity and hours components. This analysis confirms the conclusion from the discussion here that changes in labor productivity, rather than in labor input, are the key driver of low-frequency movements in real GDP growth.

delay explains their persistent overestimation of GDP growth since the recession.

It is interesting to compare the results of our decomposition exercise to similar approaches in the literature, in particular, Gordon (2010, 2014a) and Reifschneider et al. (2013). Like us, they specify a state-space model with a common cyclical component and use the output identity to decompose the long-run growth rate of GDP into underlying drivers. A key difference resides in the Bayesian estimation of the model, which enables us to impose a conservative prior on the variance of the long-run growth component that helps avoid overfitting the data. Furthermore, the inclusion of SV in the cyclical component helps to prevent unusually large cyclical movements from contaminating the long-run estimate. Another important difference is that we use a larger amount of information, including key cyclical indicators like industrial production, sales, and business surveys, which are generally not included in a production function approach. This allows us to retrieve a timely and precise estimate of the cyclical component and, as a consequence, reduce the uncertainty that is inherent in any trend-cycle decomposition of the data, as discussed in section IVF. As a result, we obtain a substantially less pessimistic estimate of the long-run growth of GDP than these studies in the latest part of the sample. For instance, Gordon (2014a) reports a long-run GDP growth estimate below 1% for the end of the sample, whereas our median estimate stands at around 2%.<sup>36</sup>

#### A. International Evidence

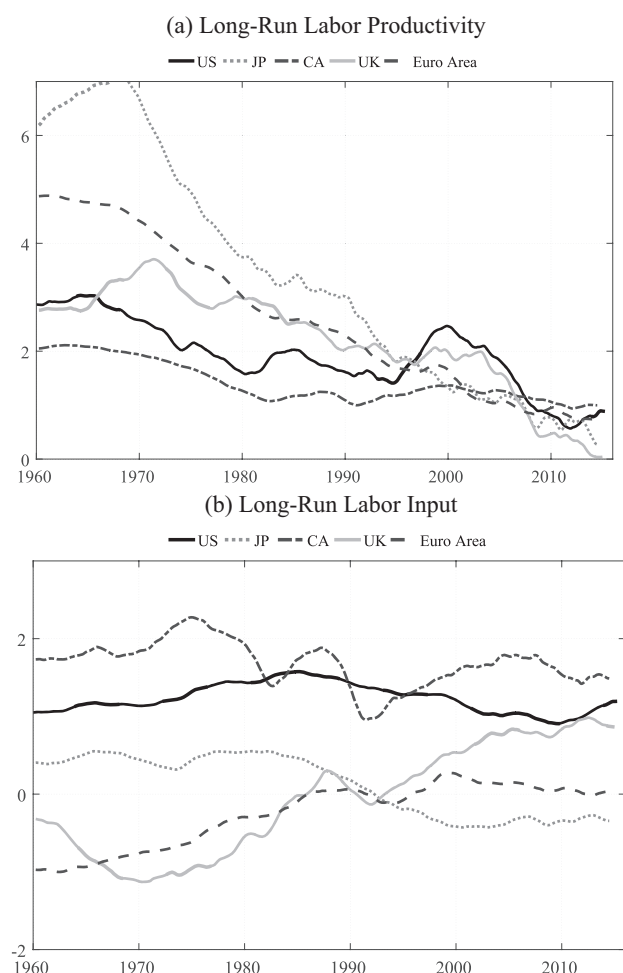
To gain an international perspective on our results, we estimate the DFM for the other G7 economies and perform the decomposition exercise for each of them.<sup>37</sup> The median posterior estimates of the labor productivity and labor input trends are displayed in figure 5. Labor productivity, displayed in figure 5a, again plays the key role in determining movements in long-run growth. In the Western European economies and Japan, the elevated growth rates of labor productivity prior to the 1970s reflect the rebuilding of the capital stock from the destruction from World War II and ended as these economies converged toward U.S. levels of output per capita. The labor productivity profile of Canada broadly follows that of the United States, with a slowdown in the 1970s and a temporary mild boom during the late 1990s. Interestingly, this acceleration in the 1990s did not occur in

<sup>36</sup> The results for a bivariate model of GDP and unemployment, which we have discussed in section IVF, show that the current long-run growth estimate is 1.3%, close to Gordon (2014a).

<sup>37</sup> Details on the specific data series used for each country are available in online appendix E. For hours, we again follow the methodology of Ohanian and Raffo (2012). In the case of the United Kingdom, the quarterly series for hours displays a drastic change in its stochastic properties in the early 1990s owing to a methodological change in the construction by the ONS, as confirmed by the ONS LFS manual. We address this issue by using directly the annual series from the Conference Board's database, which requires an appropriate extension of equation (9) to annual variables. To avoid weak identification of  $h_t$  for the United Kingdom, we truncate our prior on its variance to discard values that are larger than twice the maximum posterior draw of the case of the other countries.



FIGURE 5.—DECOMPOSITION FOR OTHER ADVANCED ECONOMIES



(a) The posterior median of long-run labor productivity across advanced economies. (b) The corresponding estimates of long-run total hours worked. In both panels, "Euro Area" represents a weighted average of Germany, Italy, and France.

Western Europe and Japan.<sup>38</sup> It is interesting to note that the two countries that experienced a more severe financial crisis, the United States and the United Kingdom, appear to be the ones with greatest declines in productivity since the early 2000s, similar to the evidence documented in Reinhart and Rogoff (2009).

Figure 5b displays the movements in long-run hours worked identified by equation (10). The contribution of this component to overall long-run output growth varies considerably across countries. However, within each country, it is more stable over time than the productivity component, which is in line with our findings for the United States. Indeed, the extracted long-run trend in total hours includes various potentially offsetting forces that can lead to changes in long-run output growth. In any case, the results of our decomposition exercise indicate that after using the DFM to remove business cycle variation in hours and output, the decline in long-run GDP growth that has been observed in

the advanced economies since the early 2000s is entirely accounted for by a decline in the labor productivity trend. Finally, it is interesting to note that for the countries in the sample, long-run productivity growth appears to converge in the cross section, while there is no evidence of convergence in the long-run growth of hours.

## VI. Conclusion

The sluggish recovery from the Great Recession has raised the question of whether the long-run growth rate of U.S. real GDP is now lower than it has been on average over the postwar period. We have presented a dynamic factor model that allows for changes in both long-run GDP growth and stochastic volatility. Estimating the model with Bayesian methods, we provide evidence that long-run growth of U.S. GDP displays a gradual decline after the turn of the century, moving from its peak of 3.5% to about 2% in 2015. Using real-time vintages of data, we demonstrate the model's ability to track GDP in a timely and reliable manner. By summer 2010, the model would have concluded that a significant decline in long-run growth was behind the slow recovery, therefore substantially improving the real-time tracking of GDP by explicitly taking into account the uncertainty surrounding long-run growth. Finally, we discuss the drivers of movements in long-run output growth through the lens of our model by decomposing it into the long-run growth rates of labor productivity and labor input. Using data for both the United States and other advanced economies, our model points to a global slowdown in labor productivity as the main driver of weak growth in recent years, extending the narrative of Fernald (2014) to other economies. Studying the deep causes of the secular decline in growth is an important priority for future research.

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<sup>38</sup> On the lost decade in Japan, see Hayashi and Prescott (2002). Gordon (2004) examines the absence of the IT boom in Europe.

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